4) there are no free charges, $\rho = 0$,

Maxwell's equations are satisfied at any point in the medium

$$\nabla \times H = -\frac{\partial B}{\partial t} \quad \text{(a)}$$

$$\nabla \times E = \frac{\partial D}{\partial t} \quad \text{(b)}$$

$$B = \mu_0(H + M) \quad \text{(c)}$$

$$D = \epsilon_0 E + P \quad \text{(d)}$$

$$\text{div } D = 0, \quad \text{div } B = 0 \text{ (e)}. \qquad (10)$$

Let the medium be excited sinusoidally with respect to time, the excitation angular frequencies being ω_1, ω_2, ω_k. Their sideband and higher harmonics

$$\sum_{i=1}^{k} n_i \omega_i,$$

are generated by the nonlinearity.

At any point in the medium, the electric field E, whose frequency is $n_1\omega_1 + n_2\omega_2 + \cdots + n_k\omega_k$, can be expanded by the multiple Fourier Series

$$E(r) = \sum_{n_k=-\infty}^{\infty} \cdots \sum_{n_1=-\infty}^{\infty} E_{n_1 n_2 \cdots n_k}(r) \cdot e^{j(n_1\omega_1 + n_2\omega_2 + \cdots n_k\omega_k)t}, \qquad (11)$$

where r is the vector representing the position of observation from the origin.

Since E is real, it follows that

$$E_{n_1 n_2 \cdots n_k} = E^*_{n_1 n_2 \cdots n_k},$$

$$E_{-n_1 - n_2 \cdots - n_k} = E^*_{n_1 n_2 \cdots n_k}. \qquad (12)$$

These results are also valid for magnetic field H, electric polarization P, current density J, and magnetization M.

From (10a) and (10b),

$$\nabla \times E_{n_1 n_2 \cdots n_k} = -j\mu_0(n_1\omega_1 + n_2\omega_2 + \cdots + n_k\omega_k) \cdot (H_{n_1 n_2 \cdots n_k} + M_{n_1 n_2 \cdots n_k}) \qquad (13)$$

$$\nabla \times H_{n_1 n_2 \cdots n_k} = j(n_1\omega_1 + n_2\omega_2 + \cdots + n_k\omega_k) \cdot (\epsilon_0 E_{n_1 n_2 \cdots n_k} + P_{n_1 n_2 \cdots n_k}). \qquad (14)$$

Making the vector product $E_{n_1 n_2 \cdots n_k} \times H_{n_1 n_2 \cdots n_k}$ by (13) and (14) and applying the vector formula

$$\nabla \cdot (A \times B) = B \cdot \nabla \times A - A \cdot \nabla \times B;$$

and summing up with to the frequencies, from $-\infty$ to $+\infty$, k independent relations are obtained as follows,

$$\nabla \cdot \sum_{n_k=-\infty}^{\infty} \cdots \sum_{n_1=-\infty}^{\infty} \frac{n_i E_{n_1 n_2 \cdots n_k} \times H^*_{n_1 n_2 \cdots n_k}}{n_1\omega_1 + n_2\omega_2 + \cdots + n_k\omega_k} = -j \sum_{n_k=-\infty}^{\infty} \cdots \sum_{n_1=-\infty}^{\infty} n_i M_{n_1 \cdots n_k} \cdot H^*_{n_1 n_2 \cdots n_k}$$

$$+ j \sum_{n_k=-\infty}^{\infty} \cdots \sum_{n_1=-\infty}^{\infty} n_i P^*_{n_1 n_2 \cdots n_k} \cdot E_{n_1 n_2 \cdots n_k}$$

$$i = 1, 2, \cdots, k. \qquad (15)$$

From assumption (2), $\sigma = 0$, and (3), the functions describing the relations between H and M and between E and P are single valued, *i.e.*,

$$H = H(M), \quad E = E(P) \cdots \text{Single Valued.} \quad (16)$$

The energy supplied to the material of unit volume so as to cause magnetic polarization M, is given by the integral (16), H being a function of M,

$$\int_0^M H(M) dM. \qquad (17)$$

Since H is single valued, integral (17) is independent of the integrating path. The integral along the closed path vanishes, since the medium is lossless. The magnetic polarization M is expanded in the multiple Fourier Series at any point in the medium as,

$$M = \sum_{n_k=-\infty}^{\infty} \cdots \sum_{n_1=-\infty}^{\infty} M_{n_1 n_2 \cdots n_k} e^{j(n_1 x_1 + n_2 x_2 + \cdots n_k x_k)}$$

$$x_i = \omega_i t \quad (i = 1, 2, \cdots k). \qquad (18)$$

The coefficient $M_{n_1 n_2 \cdots n_k}$ in (18) is expressed by,

$$M_{n_1 n_2 \cdots n_k} = \frac{1}{(2\pi)^k} \int_0^{2\pi} dx_k \int_0^{2\pi} dx_{k-1} \cdots \int_0^{2\pi} dx_i M(x_1 x_2 \cdots x_k) e^{-j(n_1 x_1 + n_2 x_2 + \cdots + n_k x_k)}$$

$$M_{n_1 n_2 \cdots n_k} = M^*_{-n_1 - n_2 \cdots - n_k},$$

$$M_{-n_1 - n_2 \cdots - n_k} = M^*_{n_1 n_2 \cdots n_k}, \qquad (19)$$

which may be rewritten as

$$H = H[M(x_1, x_2, \cdots x_k)] = H(x_1, x_2, \cdots, x_k). \qquad (20)$$

Likewise, H may be expressed by multiple Fourier Series.

By using the condition that $H(M)$ is single valued, the following equations are obtained.

$$\sum_{n_k=-\infty}^{\infty} \cdots \sum_{n_1=-\infty}^{\infty} j n_i H^*_{n_1 n_2 \cdots n_k} \cdot M_{n_1 n_2 \cdots n_k} = 0$$

$$(i = 1, 2, \cdots k). \qquad (21)$$

For P and E

$$\sum_{n_k=-\infty}^{\infty} \cdots \sum_{n_1=-\infty}^{\infty} j n_i P^*_{n_1 n_2 \cdots n_k} \cdot E_{n_1 n_2 \cdots n_k} = 0$$

$$(i = 1, 2, \cdots, k). \qquad (22)$$

Putting (21), (22) and $\sigma = 0$, into (15), k independent equations are obtained in the lossless reciprocal medium excited by k fundamental frequencies. Thus

$$\nabla \cdot \sum_{n_k=-\infty}^{\infty} \sum_{n_{k-1}=-\infty}^{\infty} \cdots \sum_{n_1=-\infty}^{\infty} \frac{n_1 E_{n_1 n_2 \cdots n_k} \times H^*_{n_1 n_2 \cdots n_k}}{n_1\omega_1 + n_2\omega_2 + \cdots + n_k\omega_k} = 0$$

$$\nabla \cdot \sum_{n_k=-\infty}^{\infty} \sum_{n_{k-1}=-\infty}^{\infty} \cdots \sum_{n_1=-\infty}^{\infty} \frac{n_2 E_{n_1 n_2 \cdots n_k} \times H^*_{n_1 n_2 \cdots n_k}}{n_1\omega_1 + n_2\omega_2 + \cdots + n_k\omega_k} = 0 \qquad (23)$$

$$\vdots$$

$$\nabla \cdot \sum_{n_k=-\infty}^{\infty} \sum_{n_{k-1}=-\infty}^{\infty} \cdots \sum_{n_1=-\infty}^{\infty} \frac{n_k E_{n_1 n_2 \cdots n_k} \times H^*_{n_1 n_2 \cdots n_k}}{n_1\omega_1 + n_2\omega_2 + \cdots + n_k\omega_k} = 0.$$

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H. IWASAWA
Dept. of Wireless Engrg.
Okubo Works, Kobe Kogyo Corp.
Okubo, Akashi, Japan

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A Method of Improving Isolation in Multi-Channel Waveguide Systems*

In microwave measurements, frequent use is made of "two channel" or multiple channel systems, in order to obtain isolation from amplitude fluctuations in the generator output and/or various other benefits. In such systems it is generally required that the signal delivered to one channel be independent of changes in loading, etc., in the second channel, and a common problem is that of achieving or determining that an adequate degree of isolation exists between the different channels. It is the purpose of this letter to describe briefly procedures for obtaining and recognizing this condition.

Consider first the three arm junction of Fig. 1, where the generator feeds arm 1. The required condition for isolation is that

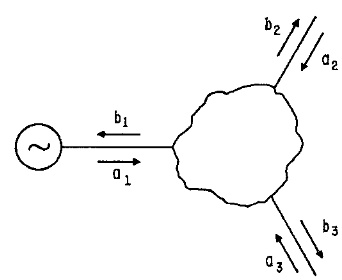

Fig. 1—Three arm waveguide junction

* Received by the PGMTT, March 10, 1960.

the ratio of the complex amplitudes of the emergent waves from arms 2 and 3 (b_2, and b_3), be constant:

$$\frac{b_2}{b_3} = K \qquad (1)$$

where K is a complex constant of finite magnitude and different from zero.

It may be noted that this condition does not actually insure that b_2, for example, will be independent of changes in the loading on arm 3. However, it does require, if a change in b_2 occurs, a simultaneous change in b_3 such that the net result is equivalent to an amplitude variation and/or phase shift in the generator to which the system is presumably insensitive.

In terms of the incident wave amplitudes (a_1, a_2, a_3), b_2 and b_3 are given by:

$$\begin{aligned} b_2 &= S_{21}a_1 + S_{22}a_2 + S_{23}a_3 \\ b_3 &= S_{31}a_1 + S_{32}a_2 + S_{33}a_3, \end{aligned} \qquad (2)$$

where the $S_{m,n}$ are the scattering coefficients of the junction.

Substituting (2) in (1) yields

$$(S_{21} - KS_{31})a_1 + (S_{22} - KS_{32})a_2 + (S_{23} - KS_{33})a_3 = 0. \qquad (3)$$

In practice a_1 will not vanish since this is the emergent wave from the generator, and in the general case both a_2 and a_3 will assume arbitrary and independent values. The criterion for isolation, therefore, requires that each of the coefficients of the a's vanish.

With reference to the coefficient of a_1, if S_{21} vanishes, so must S_{31} and conversely. But this would mean that no transmission would be possible between arm 1 and either of the other two arms, and consequently represents a trivial solution. Therefore,

$$\frac{S_{21}}{S_{31}} = K.$$

Substituting this result in the coefficients of a_2 and a_3 yields

$$S_{22} - \frac{S_{21}S_{32}}{S_{31}} = 0 \qquad (4)$$

$$S_{33} - \frac{S_{31}S_{23}}{S_{21}} = 0. \qquad (5)$$

Eqs. (4) and (5) give the necessary and sufficient conditions that the junction satisfy (1).

In practice these conditions are approximately satisfied by a "Magic T" whose shunt or series arm has been terminated in a matched load, or by a directional coupler. In such junctions, each of the terms S_{22}, S_{23}, S_{32}, and S_{33} ideally vanish. The degree of isolation achieved when using a non-ideal hybrid junction or directional coupler can generally be improved in the following manner.

Let tuning transformers be added to each of the arms of the junction as shown in Fig. 2. The generator is removed from arm 1 and replaced by a passive load (not necessarily matched). If transformer T_1 is now adjusted so that a null[1] obtains in arm 3 with the generator connected to arm 2, it can be shown that the reflection coefficient "looking into" arm 2 is just the left hand member of (4). Transformer T_2 is now adjusted so that this reflection coefficient vanishes, and (4) is satisfied. It may be intuitively recognized that this adjustment is independent of the subsequent adjustment of transformer T_3 since if a null exists in arm 3, the reflection coefficient observed at arm 2 will evidently be independent of the manner of terminating arm 3. The interchange of arms 2 and 3 in the above procedure satisfies (5). (If, as is frequently the case, the load terminating one of the arms, arm 3 for example, is constant or not subject to variations in impedance, then it can be shown that this second step or the adjustment of the transformer in the arm terminated by the fixed load is not required.) Once the proper adjustment of T_2 and T_3 has been realized, the adjustment of T_1 is no longer important since the conditions expressed by (4) and (5) are invariant to the adjustment of T_1. If, as an additional condition, T_1 is terminated by a load equal to the generator impedance during the tuning procedure, then b_2 and b_3 individually, as well as their ratio, will remain constant. In many applications this is of at least nominal interest.

The extension of this procedure to three or more channels is straightforward. In the four arm junction of Fig. 3, for example, let the arm which will ultimately be connected to the generator (arm 1) be connected to an arbitrary load as previously. If now by means of tuner T_1 and other "internal" tuning adjustments (which will in general be required), it is possible to achieve the condition where no coupling exists between the remaining arms (no output is observed at arms 3 and 4 with arm 2 connected to the generator, etc.) and if tuners T_2, T_3, T_4, have been adjusted so that the reflection coefficients at the respective arms vanish, then it can be shown that the desired condition obtains. The considerations involving tuner T_1, as outlined above, also continue to apply.

One possible method of obtaining such a junction is shown in Fig. 4, where tuner T_a provides the "internal" adjustment as required above.

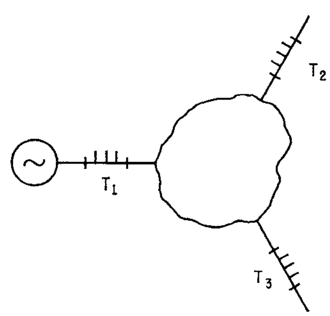

Fig. 2—Three arm junction with tuners.

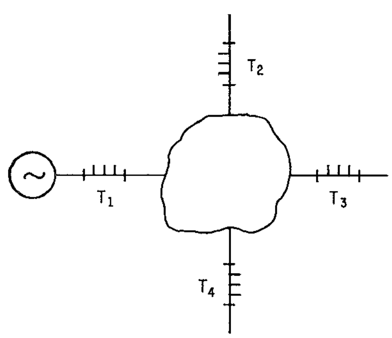

Fig. 3—Four arm junction and tuners.

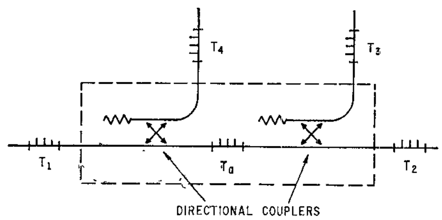

Fig. 4—Four arm junction with internal tuning.

G. F. Engen
U. S. Dept. of Commerce
Natl. Bur. of Standards
Boulder Labs.
Boulder, Col.

[1] If transformer T_1 is assumed to be dissipationless, this adjustment may be realized provided that the junction satisfies the condition

$$\left| S_{11} - \frac{S_{12}S_{31}}{S_{32}} \right| > 1.$$

A Microwave Impedance Meter Capable of High Accuracy*

Recent applications of directional couplers with auxiliary tuners for accurate VSWR and phase shift measurements have made possible a microwave impedance meter capable of high accuracy. It is similar to some bridges in that one obtains an initial detector null and a final null, before and after connecting the unknown. Both the magnitude and phase angle of the reflection coefficient of the unknown are determined in this operation, and these can be made direct reading if desired. The principle of operation is as follows.

The use of directional couplers with auxiliary tuners permits adjustment for the conditions $S_{31} = \Gamma_{2i} = 0$, whereupon

$$b_3 = C\Gamma_L. \qquad (1)$$

The symbols have the following meanings, which become clearer upon reference to Fig. 1.

S_{31} = transmission (scattering) coefficient for waves going from arm 1 to arm 3,

Γ_{2i} = reflection coefficient which would be measured "looking into" arm 2 if the generator were replaced by a passive impedance having the same impedance as the generator,

b_3 = amplitude of wave emerging from arm 3,

C = a constant which depends upon the parameters of the adjusted directional coupler-tuner combination,

* Received by the PGMTT, April 1, 1960.